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On integrable quantum group invariant antiferromagnets

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A new open spin chain hamiltonian is introduced. It is both integrable (Sklyanin's type K matrices are used to achieve this) and invariant under $\mathcal{U}_\epsilon\,(\mathrm{sl}(2))$ transformations in nilpotent irreps for $\epsilon^3 = 1$. Some considerations on the centralizer of nilpotent representations and its representation theory are also presented.

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The most direct way to get into the physics associated with quantum groups is certainly the study of quantum mechanical systems possessing a quantum group symmetry. Some examples of this kind are already known in the context of one dimensional spin chains [1]. The simplest one is the XXZ spin 1/2 chain with boundary conditions:

$$H = \sum_{i=1}^{N-1} \sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \tfrac{1}{2}(q+q^{-1})\sigma_i^z\sigma_{i+1}^z + \tfrac{1}{2}(q-q^{-1})(\sigma_1^z - \sigma_N^z), \quad (1)$$

which is invariant under $\mathcal{U}_q(\mathrm{sl}(2))$ transformations. The integrable version of the spin one Heisenberg model with non vanishing anisotropy is the Zamolodchikov–Fateev hamiltonian [2]:

$$H^{\mathrm{ZF}} = \sum_{i=1}^{N-1} H^{\mathrm{ZF}}_{i,i+1}$$

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$$
\begin{aligned}
&= \sum_{i=1}^{N-1} S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \tfrac{1}{2}(q^2 + q^{-2}) S_i^z S_{i+1}^z \\
&\quad - (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)^2 - \tfrac{1}{2}(q^2 + q^{-2})(S_i^z S_{i+1}^z)^2 \\
&\quad + (1 - q - q^{-1})[(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) S_i^z S_{i+1}^z + \leftrightarrow] \\
&\quad + [\tfrac{1}{2}(q^2 + q^{-2}) - 1][(S_i^z)^2 + (S_{i+1}^z)^2] . \qquad (2)
\end{aligned}
$$

The hamiltonian $H_{i,i+1}^{\mathrm{ZF}}$ is given by the logarithmic derivative of the spin one R-matrix $R^{(s=1)}(u)$ of the affine Hopf algebra $\mathcal{U}_q(\widehat{\mathrm{sl}(2)})$. In order to make the ZF hamiltonian invariant under global $\mathcal{U}_q(\mathrm{sl}(2))$ transformations the following boundary term should be added:

$$
H^{\mathrm{B}} = \tfrac{1}{2}(q^2 + q^{-2})(S_N^z - S_1^z) . \qquad (3)
$$

Integrability in a box requires that the Sklyanin reflection operators $K_{\pm}(u)$, which describe the scattering with the wall, obey the relations [3]:

$$
\begin{aligned}
&R_{12}(u - v) K_-^{(1)}(u) R_{12}(u + v) K_-^{(2)}(v) \\
&\quad = K_-^{(2)}(u) R_{12}(u + v) K_-^{(1)}(u) R_{12}(u - v) . \qquad (4)
\end{aligned}
$$

For the quantum group invariant hamiltonian $H^{\mathrm{ZF}} + H^{\mathrm{B}}$ it is not hard to prove [4] that

$$
[H^{\mathrm{ZF}} + H^{\mathrm{B}}, t(u)] = 0 , \qquad (5)
$$

where the box transfer matrix $t(u)$ is defined by

$$
t(u) = \mathrm{Tr}(K_+(u) T(u) K_-(u) T^{-1}(-u)) \qquad (6)
$$

with K_- satisfying (4) for R the spin one R-matrix of $\mathcal{U}_q(\widehat{\mathrm{sl}(2)})$ and $K_+(u) = K_-(-u-\eta)$, $q = \exp \eta$. From (5) the integrability of the hamiltonian $H^{\mathrm{ZF}} + H^{\mathrm{B}}$ follows. The monodromy matrix $T(u)$ in (6) is the one defined by the $s = 1$ quantum R-matrix of $\mathcal{U}_q(\widehat{\mathrm{sl}(2)})$.

In ref. [5] a one parameter family of integrable deformations of (2) was defined for $q = \epsilon$, $\epsilon^3 = 1$:

$$
\begin{aligned}
H(\lambda) &= 2(\epsilon - \epsilon^{-1}) \sum_{i=1}^{N-1} \tfrac{1}{2}(\lambda\epsilon + \lambda^{-1}\epsilon^{-1})(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) \\
&\quad - \tfrac{1}{2} S_i^z S_{i+1}^z - (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)^2 + \tfrac{1}{2}(S_i^z S_{i+1}^z)^2 \\
&\quad + [\tfrac{1}{2}(\lambda\epsilon + \lambda^{-1}\epsilon^{-1}) + \omega][(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) S_i^z S_{i+1}^z + \leftrightarrow] \\
&\quad - \tfrac{3}{2}[(S_i^z)^2 + (S_{i+1}^z)^2] \\
&\quad - \frac{\lambda\epsilon - \lambda^{-1}\epsilon^{-1}}{2(\epsilon - \epsilon^{-1})}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)(S_i^z + S_{i+1}^z) \\
&\quad + \frac{\lambda^2\epsilon^{-1} - \lambda^{-2}\epsilon}{2(\epsilon - \epsilon^{-1})}(S_i^z + S_{i+1}^z)
\end{aligned}
$$

$$\equiv \sum_{i=1}^{N-1} H(\lambda)_{i,i+1}, \tag{7}$$

$$\omega(\lambda) = \sqrt{(\lambda-\lambda^{-1})(\lambda\epsilon^{-1}-\lambda^{-1}\epsilon)}.$$

The hamiltonian $H(\lambda)_{i,i+1}$ is (up to a constant factor) the logarithmic derivative of the quantum R-matrix $R^{(\lambda)}(u)$ which intertwines nilpotent representations of $\mathcal{U}_q(\widehat{\mathrm{sl}(2)})$ for $q = \epsilon$ [5,6]. Let us recall that nilpotent representations of $\mathcal{U}_\epsilon(\widehat{\mathrm{sl}(2)})$ ($\epsilon^3 = 1$) are three dimensional irreducible representations transforming under the central Hopf subalgebra as $E^3 = F^3 = 0$, $K^3 = \lambda^3$, with λ a generic complex number. For $\lambda = \epsilon^2$, which corresponds to the spin one representation, $H(\lambda)$ coincides with $2(\epsilon - \epsilon^{-1})H^{\mathrm{ZF}}$ for $q = \epsilon$.

The problem we want to address in this lecture is the definition of an integrable and quantum group invariant version of (7). Quantum group invariance is easily obtained adding to (7) the boundary term:

$$H^{\mathrm{B}}(\lambda) = \omega^2(S_1^z - S_N^z). \tag{8}$$

The hamiltonian $H(\lambda) + H^{\mathrm{B}}(\lambda)$ coincides, for $\lambda = \epsilon^2$, with $2(\epsilon-\epsilon^{-1})(H^{\mathrm{ZF}} + H^{\mathrm{B}})$, which is already a good indication concerning integrability. However, to attain it, we need to check for $H(\lambda) + H^{\mathrm{B}}(\lambda)$ the equivalent of eq. (5) with K now being a solution to (4) for R the quantum nilpotent $R^\lambda(u)$-matrix. The K-matrices for the nilpotent $R^\lambda(u)$-matrix are:

$$\begin{aligned}
K_-(u) &= \frac{1}{\sinh\alpha_- \sinh(\alpha_- - \eta)} \\
&\quad\times \mathrm{diag}(\sinh(u+\alpha_-)\sinh(u+\alpha_- - \eta), \\
&\qquad -\sinh(u-\alpha_-)\sinh(u+\alpha_- - \eta), \\
&\qquad \sinh(u-\alpha_-)\sinh(u-\alpha_- + \eta)), \\
K_+(u) &= \mathrm{diag}(\sinh(u+\eta-\alpha_+)\sinh(u-\alpha_+ - \eta), \\
&\qquad -\sinh(u+\eta+\alpha_+)\sinh(u-\alpha_+ - \eta), \\
&\qquad \sinh(u+\alpha_+)\sinh(u+\alpha_+ + \eta)),
\end{aligned} \tag{9}$$

with $\alpha_\pm$ free parameters and $\eta = 2\pi\mathrm{i}/3$. Note that these matrices possess precisely the same form as those used in ref. [4] to prove the integrability of the Zamolodchikov–Fateev spin one chain with boundary terms. Using these K-matrices we derive for $H(\lambda) + H^{\mathrm{B}}(\lambda)$ the integrability condition (5) by showing that $H(\lambda) + H^{\mathrm{B}}(\lambda)$ is proportional to the second derivative, at the point $u = 0$, of the box transfer matrix $t(u)$, for $\alpha_\pm = \infty$. Notice the difference with the ZF case, where the hamiltonian is given by the first logarithmic derivative of $t(u)$. The reasons are that in our case, as can be seen from (9), $t(0) = \mathrm{Tr}\, K_+(0)$ becomes zero, and that $\mathrm{Tr}(K_+^{(0)}(0)H(\lambda)_{N0}) \propto \mathbf{1}^{(N)}$.

The nice thing about a quantum group invariant hamiltonian is that most of the properties of the spectrum can be directly derived from representation theory. So, for instance, for the hamiltonian (1) we know that each energy eigenvalue is associated with a given spin-j irrep of $\mathcal{U}_q(\mathrm{sl}(2))$ and that it would be $(2j+1)$-fold degenerate. The different irreps that can appear in the spectrum are the ones obtained by decomposing $\bigotimes^N V^{1/2}$. Moreover the highest weight vectors transforming in a different representation with the same j will define irreducible representations of the centralizer of $\mathcal{U}_q(\mathrm{sl}(2))$, which for spin 1/2 is given by the Hecke algebra. In the massless phase ($|q| = 1$), the previous results provide, together with the systematic use of the finite size technique [7], the basis for the quantum group interpretation of conformal field theories [8]. A similar study can now be done for the hamiltonian $H(\lambda) + H^{\mathrm{B}}(\lambda)$ with the new features being associated to the peculiarities of the representation theory at roots of unity.

In what follows we will concentrate our analysis on the structure of the centralizer for nilpotent representations of $\mathcal{U}_\epsilon(\mathrm{sl}(2))$. Given a nilpotent representation V^λ we define the centralizer $C_N^\lambda(\epsilon)$ as the algebra of endomorphisms $g : \bigotimes^N \to \bigotimes^N V^\lambda$ commuting with the quantum group action. To get the generators of $C_N^\lambda(\epsilon)$ we first define the "braiding limit" of the quantum R-matrix $R^{\lambda\lambda}(u)$ of the affine Hopf algebra $\mathcal{U}_\epsilon(\widehat{\mathrm{sl}(2)})$ as follows:

$$R_\pm^\lambda = \lim_{u\to\pm\infty} R^{(\lambda,\lambda)}(u)_{r_1 r_2}^{r_1' r_2'}\, \mathrm{e}^{u(r_1 - r_2')} . \tag{10}$$

Elements in $C_N^\lambda(\epsilon)$ are then generated by

$$g_i^\pm = \mathbf{1} \otimes \cdots \otimes (R_\pm^\lambda)_{i,i+1} \otimes \cdots \otimes \mathbf{1}, \quad i = 1, \dots, N-1 . \tag{11}$$

Based on the spectral decomposition of $R_\pm^\lambda$ we will assume that the set of generators g_i is complete. In order to get some insight into the structure of the centralizer we will first consider the case $\epsilon^4 = 1$. In this case the nilpotent representations are two dimensional and the "braiding limit" R-matrix is given by:

$$\begin{aligned} R^\lambda &= \begin{pmatrix} \lambda & & & \\ & \lambda - \lambda^{-1} & 1 & \\ & 1 & 0 & \\ & & & -\lambda^{-1} \end{pmatrix} \\ &= \sigma^+ \otimes \sigma^- + \sigma^- \otimes \sigma^+ + \tfrac{1}{2}\lambda^{-1}(\sigma^z \otimes \mathbf{1}) + \tfrac{1}{2}\lambda(\mathbf{1} \otimes \sigma^z) + \tfrac{1}{2}(\lambda - \lambda^{-1})\mathbf{1} \otimes \mathbf{1} . \end{aligned} \tag{12}$$

This R-matrix has two eigenvalues, λ and $-\lambda^{-1}$. The generators g_i satisfy the Hecke relation:

$$g_i^2 = (\lambda - \lambda^{-1}) g_i + \mathbf{1} . \tag{13}$$

This means that the centralizer $C_N^\lambda(\epsilon)$ in the case $\epsilon = \mathrm{e}^{\pi \mathrm{i}/2}$ gives us a representation of the Hecke algebra $H_N(\lambda^2)$. It is well known that for generic q the

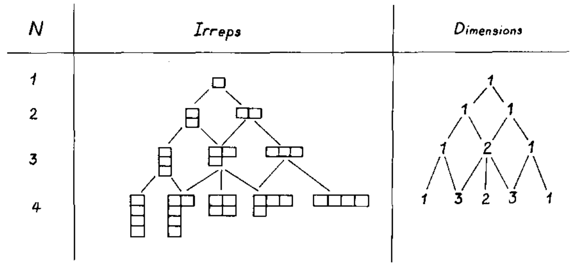


Fig. 1.

irreducible representations of $H_N(q)$ are in one to one correspondence with irreps of S_N, see fig. 1. So we may ask which representations we get from the centralizer $C_N^{\lambda}(\epsilon)$. At this point it is worthwhile to recall that the centralizer $C_N^{1/2}(q)$ for the spin 1/2 representation of $\mathcal{U}_q(\mathrm{sl}(2))$ is the quotient of a Hecke algebra $H_N(q)$ by the relation

$$g_i g_{i+1} g_i + g_i g_{i+1} + g_{i+1} g_i + g_i + g_{i+1} + 1 = 0, \tag{14}$$

which in turn is equivalent to reducing the allowed Young tableaux to those with at most two rows.

In this case the $R^{1/2}$-matrix which intertwines two spin 1/2 irreducible representations of $\mathcal{U}_q(\mathrm{sl}(2))$ is given by:

$$R^{1/2} = \begin{pmatrix} q & & & \\ & 0 & q^{1/2} & \\ & q^{1/2} & q-1 & \\ & & & q \end{pmatrix}. \tag{15}$$

This R-matrix has also two eigenvalues, -1 and q, but the main difference with respect to the R-matrix (12) is that for (12) the multiplicities of the eigenvalues are 2 and 2, while for (15) they are 1 and 3. The latter fact can be understood from the decomposition rule $\frac{1}{2}\otimes\frac{1}{2} = 0\oplus 1$ (irrep 0 has dimension 1 and irrep 1 is three dimensional). More generally we see that condition (14) imposes a one to one relation between the irreps of the centralizer $C_N^{1/2}(q)$ and the decomposition into irreps of $\mathcal{U}_q(\mathrm{sl}(2))$ of $\bigotimes^N V^{1/2}$. All this means that the Brauer–Weyl theory also applies to the spin 1/2 representation of $\mathcal{U}_q(\mathrm{sl}(2))$.

For the centralizer $C_N^{\lambda}(\epsilon)$ we now try to follow the same steps, namely to see which are the allowed Young diagrams in fig. 1 according to the decomposition rules of nilpotent irreps. It was shown in refs. [9,10] that the decomposition rules of nilpotent irreps for generic values of λ are given by:

$$\lambda\otimes\lambda = \bigoplus_{i=0}^{N'-1} \lambda^2\epsilon^{-2i}, \tag{16}$$

where $\epsilon^N = 1$ ($N' = N$ for N odd and $N' = N/2$ for N even). In the case

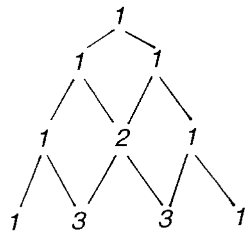
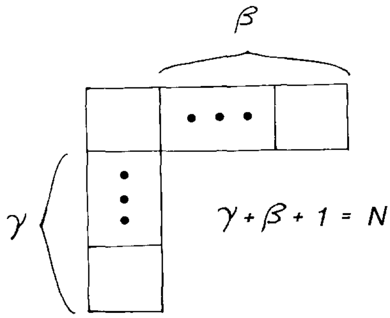


Fig. 2.

of $\epsilon = e^{i\pi/2}$ eq. (16) explains the multiplicities (2,2) of the eigenvalues of the R^λ-matrix (12), since $\lambda \otimes \lambda = \lambda^2 \oplus (-\lambda^2)$ and both λ^2 and $-\lambda^2$ have dimension 2. Moreover the generators constructed out from R^λ satisfy instead of relation (14) the following one:

$$e_i^- e_{i+2}^- e_{i+1}^+ e_i^+ e_{i+2}^+ = e_i^- e_{i+2}^- e_{i+1}^- e_i^+ e_{i+2}^+ = 0, \tag{17}$$
$$e_i^\pm \equiv (1 \pm \lambda^{\pm 1} R_{i,i+1}^\lambda)/(1 + \lambda^{\pm 2}),$$

which implies that the allowed Young diagrams, in the nilpotent case, are those of "corner" type:



(18)

Other generalizations of Hecke can be seen in ref. [11]. The Bratelli diagram describing the centralizer $C_N^\lambda(\epsilon = e^{i\pi/2})$ is that in fig. 2. We notice that the Young diagrams of the type (18) are precisely the only ones that contribute to the Alexander–Conway polynomial as shown by Jones in ref. [12]. We would also like to mention that the R-matrix (12) coincides with the intertwiner R-

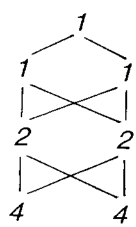


Fig. 3.

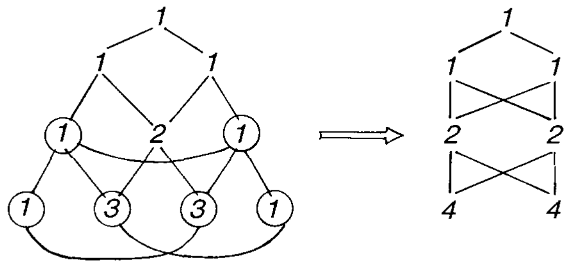


Fig. 4.

matrix for the fundamental representation of $\mathcal{U}_q(\mathrm{sl}(1,1))$ (with q replaced by λ), which was used in ref. [13] in order to construct the Alexander polynomial. It has also been found in the context of boson representations of $\mathcal{U}_q(\mathrm{sl}(2))$ [14]. All this seems to indicate alternative descriptions of the nilpotent irreps of $\mathcal{U}_\epsilon(\mathrm{sl}(2))$ for $\epsilon = e^{i\pi/2}$.

Coming back to our problem, we can now compare the Bratelli diagram in fig. 2 with the one we derive from the decomposition rule (16) in the case of $\epsilon = e^{i\pi/2}$, shown in fig. 3. It is then clear that the diagrams in figs. 2 and 3 can be related under some identifications, as in fig. 4. We now face two possibilities, either

- the set of generators g_i given by (11) is not complete in the sense that the centralizer $C_N^\lambda(\epsilon)$ is bigger, or
- the centralizer $C_N^\lambda(\epsilon)$ is nothing but the one generated by the g_i's with Bratelli diagram given by that in fig. 2 and then the Brauer–Weyl theory is not working in the standard way for nilpotent representations.

We believe that the correct possibility is the last one and we shall present computational evidence for this.

We shall consider the next non-trivial case, $\epsilon^3 = 1$; the Bratelli diagram for the centralizer is given in fig. 5. Let us compare for instance level 3 of fig. 5 with the decomposition $V^\lambda \otimes V^\lambda \otimes V^\lambda$ depicted in fig. 6. The basis of V^λ is $\{e_i\}_{i=0}^2$ and $M(e_{r_1} \otimes e_{r_2} \otimes e_{r_3}) = r_1 + r_2 + r_3$. In the figure each dot stands for one of the linearly independent states for each value of M. Dots linked by vertical

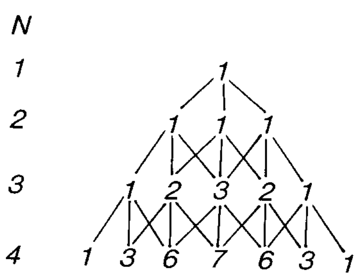


Fig. 5.

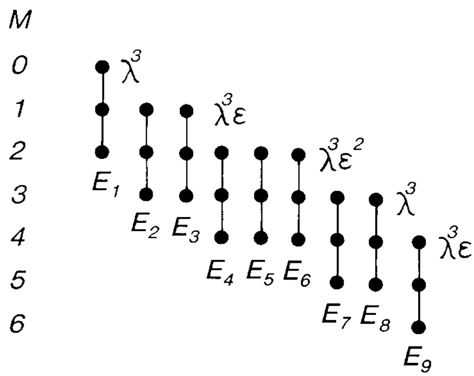


Fig. 6.

lines are connected by the action of the quantum group generators on the space $V^\lambda \otimes V^\lambda \otimes V^\lambda$, and so they have the same energy eigenvalue. They also share the same eigenvalue of the quantum group generator $\Delta^{(3)}(K)$, which is also given in the figure. We realize that the different irreps appearing in fig. 5 are one to one related with sets of irreps in fig. 6 possessing the same value of M. In fact it can be explicitly checked that the "braiding" transformations g_i defined by (11) close in the subspace defined by the same value of M. From this we can conclude that if $C_N^\lambda(\epsilon)$ is generated by the g_i's then Brauer–Weyl theory cannot be directly applied to the case of nilpotent irreps. Certainly this result does not rule out the possibility of additional generators; however, the explicit analysis of the spectrum of the hamiltonian $H(\lambda) + H^{\mathrm{B}}(\lambda)$, presented in the first part of this lecture, seems to indicate that this is not the case.

For $\epsilon^3 = 1$ and a chain of three sites the dependences on λ of the energy eigenvalues $E_1, \dots, E_9$ (see fig. 6) of $H(\lambda) + H^{\mathrm{B}}(\lambda)$ are given in fig. 7. By direct inspection of this figure we see that the energy eigenvalues corresponding to the same eigenvalue of M have a similar behaviour. It is worth mentioning that the Bratelli diagrams in figs. 2 and 5 can be derived from a modification of the decomposition rule (16). Indeed if we supplement the irrep λ with a new quantum number $n \in \mathbb{N}$, and considering the fusion rule

$$(\lambda_1, n_1) \otimes (\lambda_2, n_2) = \bigoplus_{r=0}^{N'-1} (\lambda_1 \lambda_2 \epsilon^{-2r}, n_1 + n_2 + r) \tag{19}$$

we then obtain for $N' = 2$ and 3 the Bratelli diagrams of figs. 2 and 5, respectively.

This new quantum number n is quite likely the Casimir of an algebra whose representations are identical to the nilpotent irreps of $\mathcal{U}_\epsilon(\mathrm{sl}(2))$. This is indeed the case of $N' = 2$ and ∞, where this algebra is $\mathcal{U}_q(\mathrm{gl}(1,1))$ [15] and $\mathcal{U}(\mathrm{h}_4)$ [16], respectively.

Summarizing the content of this lecture:

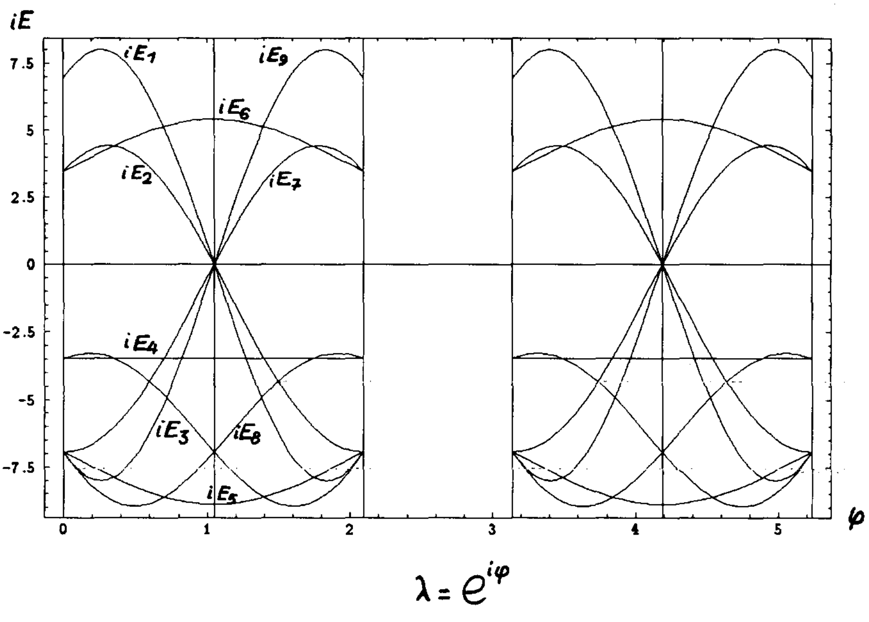


Fig. 7.

(i) We have obtained an integrable quantum group invariant spin chain hamiltonian for nilpotent representations of $\mathcal{U}_q(\mathrm{sl}(2))$ at roots of unity.

(ii) We have defined the centralizer for nilpotent representations $C_N^\lambda(\epsilon)$ and studied its representation theory. It turns out that the irreps of the centralizer generated by the g_i's in eq. (11) are one to one related (in the case $\epsilon^4 = 1$) to irreps of S_N characterized by "corner"-type Young diagrams.

Many questions remain open. Among them it would be interesting to provide a proof that $C_N^\lambda(\epsilon)$ is in fact generated by the "braiding" limit (10) of the quantum R-matrix $R^{\lambda\lambda}(u)$, and to generalize to this case the Brauer–Weyl theory. From a more speculative point of view the situation concerning the centralizer we are facing here is strongly reminiscent of the existence in CFT of extensions of chiral algebras [17].

A more detailed presentation of the content of this lecture is at present in preparation [18].

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